Problem 2.8

A mass m has velocity v_o at time $t = 0$ and coasts along the x axis in a medium where the drag force is $F(v) = -cv^{3/2}$. Use the method of Problem 2.7 to find v in terms of the time t and the other given parameters. At what time (if any) will it come to rest?

Solution

For a particle moving in one dimension, Newton's second law gives the equation of motion.

$$F = ma$$

Suppose the force is a function of velocity.

$$F(v) = m\frac{dv}{dt}$$

Substitute the given formula for $F(v)$.

$$-cv^{3/2} = m\frac{dv}{dt}$$

Separate variables.

$$-\frac{c}{m}\,dt = \frac{dv}{v^{3/2}}$$

Integrate both sides definitely, assuming that at $t = 0$ the particle has velocity v_o and at some time of interest t the particle has velocity v. Because the integration is definite, no constant of integration is needed.

$$\int_0^t -\frac{c}{m}\,dt' = \int_{v_o}^{v} \frac{dv'}{v'^{3/2}}$$

Evaluate the integrals.

$$\begin{aligned} -\frac{c}{m}t &= -2v'^{-1/2}\Big|_{v_o}^{v} \\ &= -2\left(\frac{1}{\sqrt{v}} - \frac{1}{\sqrt{v_o}}\right) \end{aligned}$$

Divide both sides by -2.

$$\frac{c}{2m}t = \frac{1}{\sqrt{v}} - \frac{1}{\sqrt{v_o}}$$

Isolate the term with v.

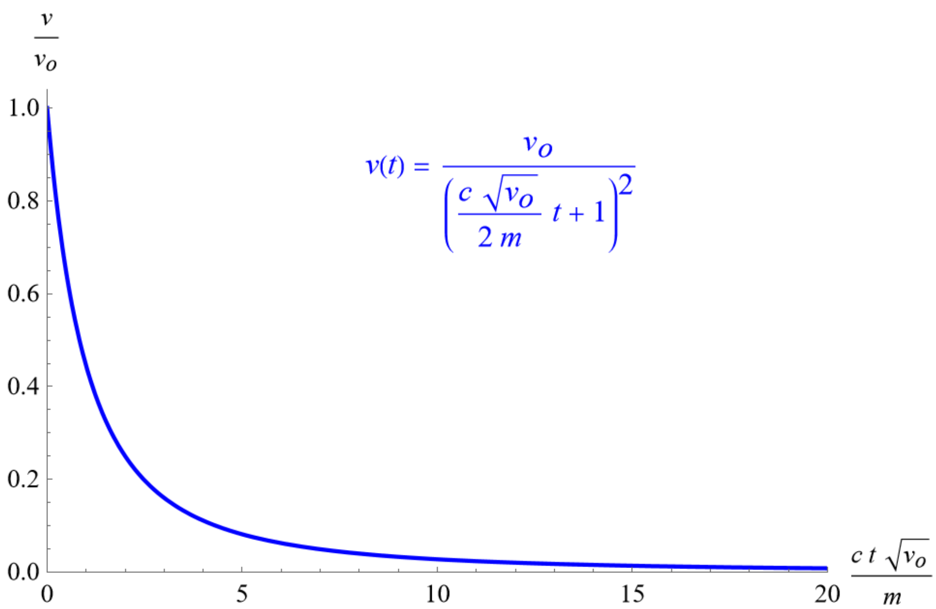
$$\begin{aligned} \frac{1}{\sqrt{v}} &= \frac{c}{2m}t + \frac{1}{\sqrt{v_o}} \\ &= \frac{ct\sqrt{v_o} + 2m}{2m\sqrt{v_o}} \end{aligned}$$

Invert both sides.

$$\sqrt{v} = \frac{2m\sqrt{v_\text{o}}}{ct\sqrt{v_\text{o}} + 2m}$$

Therefore, squaring both sides,

$$\begin{aligned} v(t) &= \frac{4m^2 v_\text{o}}{(ct\sqrt{v_\text{o}} + 2m)^2} \\ &= \frac{4m^2 v_\text{o}}{\left[2m\left(\frac{c\sqrt{v_\text{o}}}{2m}t + 1\right)\right]^2} \\ &= \frac{4m^2 v_\text{o}}{4m^2\left(\frac{c\sqrt{v_\text{o}}}{2m}t + 1\right)^2} \\ &= \frac{v_\text{o}}{\left(\frac{c\sqrt{v_\text{o}}}{2m}t + 1\right)^2}. \end{aligned}$$



The mass does not come to rest in finite time. Only in the limit as $t \to \infty$ does it stop moving.